Mapping out the quark structure of hadrons in QCD*

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In the context of QCD sum rules with non-local condensates we present a pion distribution amplitude, which is double-humped with its end-points $x \to (0,1)$ strongly suppressed, and show that it matches the CLEO experimental data on the pion-photon transition at the 1σ level accuracy, being also in compliance with the CELLO data. We also include some comments on the nucleon distribution amplitude and the nucleon evolution equation.

I. A TRIBUTE TO PROF. EFREMOV'S CELEBRATION OF HIS 70th BIRTHDAY

Prof. Efremov gives us the opportunity to point out in this Festschrift the influence of his work on our own research activities.

A. V. Efremov is one of the inventors of factorization theorems in quantum field theory that are particularly indispensable in applying perturbative QCD in inclusive [1] and exclusive reactions [2, 3] involving hadrons. Without these tools, the experimental verification of QCD would constitute an intractable task. Together with his then student A. V. Radyushkin he accomplished the factorization theorems for the meson form factors, linking diagrammatic techniques with the operator product expansion (OPE). The grounds for these works were supplied by previous investigations by Efremov [4] and Efremov and collaborators [5].

Moreover, Efremov and Radyushkin have diagonalized the anomalous dimensions matrix for meson operators (in leading order) in terms of Gegenbauer polynomials and first obtained the asymptotic distribution amplitude (DA) $\varphi(x, \mu^2 \to \infty) \to \varphi^{as}(x) = 6x(1-x)$ [3, 6].

Factorization theorems [3, 6, 7] make it possible to calculate various hard processes in QCD involving mesons, in which the meson DAs enter as the central nonperturbative input.

In the context of the present occasion, we are primarily interested in presenting recent achievements in describing the pion characteristics by mapping out its internal quark structure. A short note on the nucleon is also included.

II. NON-LOCAL CONDENSATES AND PION DISTRIBUTION AMPLITUDE

The pion DA of twist-2, $\varphi_{\pi}(x, \mu^2)$, is a gauge- and process-independent characteristic of the pion that universally specifies the longitudinal momentum xP distribution of valence quarks

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in the pion with momentum P

$$\langle 0 \mid \bar{d}(0)\gamma^{\mu}\gamma_{5}E(0,z)u(z) \mid \pi(P)\rangle\Big|_{z^{2}=0} = if_{\pi}P^{\mu} \int_{0}^{1} dx e^{ix(zP)} \varphi_{\pi}(x,\mu^{2})$$
 (2.1)

and where $E(0,z) = \mathcal{P} \exp\left[-ig_s \int_0^z t^a A_\mu^a(y) dy^\mu\right]$ is a phase factor, path-ordered along the straight line connecting the points 0 and z to preserve gauge invariance.

A. Average QCD vacuum quark virtuality λ_q^2

The pion DA encapsulates the long-distance effects and therefore reflects the nonperturbative features of the QCD vacuum. The latter can be effectively parameterized in terms of non-local condensates, as developed in [8, 9, 10] by A. Radyushkin and two of us (A.B. and S.M.). This provides a reliable method to construct hadron DAs that inherently accounts for the fact that quarks and gluons can flow through the QCD vacuum with non-zero momentum k_q . This means, in particular, that the average virtuality of vacuum quarks, $\langle k_q^2 \rangle = \lambda_q^2$ is not zero, like in the local sum-rule approach [11], but can have values in the range [12] $\lambda_q^2 = \langle \bar{q} (ig \, \sigma_{\mu\nu} G^{\mu\nu}) \, q \rangle / (2 \langle \bar{q}q \rangle) = 0.35 - 0.55 \, \text{GeV}^2$. Therefore, the non-local condensates in the coordinate representation, say, $\langle \bar{q}(0)E(0,z)q(z)\rangle$, are no longer constants, but depend on the interval z^2 in Euclidean space and decay with the correlation length $\Lambda \sim 1/\lambda_q$. Lacking an exact knowledge of non-local condensates of higher dimensionality, one has de facto to resort to specific Anzätze [13], in order to parameterize the non-local condensates. Nevertheless, it is important to stress that we were able to determine in [14] λ_q^2 directly from the CLEO data [15] within the range predicted by QCD sum rules [12] and lattice simulations [13], favoring the value $\lambda_q^2 \simeq 0.4 \, \text{GeV}^2$.

B. QCD sum rules

The distribution amplitudes $\varphi_{\pi(A_1)}(x,\mu^2)$ for the pion and its first resonance can be related to the non-local condensates by means of the following sum rule that is based on the correlator of two axial currents

$$f_{\pi}^{2}\varphi_{\pi}(x) + f_{A_{1}}^{2}\varphi_{A_{1}}(x) \exp\left\{-\frac{m_{A_{1}}^{2}}{M^{2}}\right\} = \int_{0}^{s_{\pi}^{0}} \rho^{\text{pert}}(x;s)e^{-s/M^{2}}ds + \frac{\langle \alpha_{s}GG \rangle}{24\pi M^{2}} \Phi_{G}\left(x;M^{2}\right) + \frac{8\pi\alpha_{s}\langle \bar{q}q \rangle^{2}}{81M^{4}} \sum_{i=S,V,T_{1},T_{2},T_{3}} \Phi_{i}\left(x;M^{2}\right) , \qquad (2.2)$$

where the index i runs over scalar, vector, and tensor condensates [16, 17], M^2 is the Borel parameter, and s_{π}^0 is the duality interval in the axial channel. Above, the dependence on the non-locality parameter enters on the RHS in the way exemplified by the numerically important scalar-condensate contribution

$$\Phi_{S}\left(x; M^{2}\right) = \frac{18}{\bar{\Delta}\Delta^{2}} \left\{ \theta\left(\bar{x} > \Delta > x\right) \bar{x} \left[x + (\Delta - x)\ln\left(\bar{x}\right)\right] + (\bar{x} \to x) + \theta(1 > \Delta)\theta\left(\Delta > x > \bar{\Delta}\right) \left[\bar{\Delta} + (\Delta - 2\bar{x}x)\ln(\Delta)\right] \right\}$$
(2.3)

with $\Delta \equiv \lambda_q^2/(2M^2)$, $\bar{\Delta} \equiv 1 - \Delta$ and $\bar{x} \equiv 1 - x$. In the so-called *local* approach [11], the end-point contributions $(x \to 0 \text{ or } 1)$ are strongly enhanced by $\delta(x), \delta'(x), \ldots$ because they

disregard the finiteness of the vacuum correlation length Λ by setting in Eq. (2.3) $\lambda_q^2 \to 0$ to obtain

$$\lim_{\Delta \to 0} \Phi_S\left(x; M^2\right) = 9\left[\delta(x) + \delta(1-x)\right]. \tag{2.4}$$

In contrast, taking into account the non-locality of the condensates via λ_q^2 , leads to a strong suppression of these regions. Due to the end-point suppression property, the sum rule (2.2) allows us to determine the first ten moments $\langle \xi^N \rangle_{\pi} \equiv \int_0^1 \varphi_{\pi}(x) (2x-1)^N dx$ of the pion DA and independently also the inverse moment $\langle x^{-1} \rangle_{\pi} \equiv \int_0^1 \varphi_{\pi}(x) \ x^{-1} dx$ quite accurately (see in [18, 19] for more details). The intrinsic accuracy of this procedure admits to obtain the pion DA moments with uncertainties varying in the range of 10%.

C. Models for the pion distribution amplitude

Models for the pion DA, in correspondence to the extracted moments, can be constructed

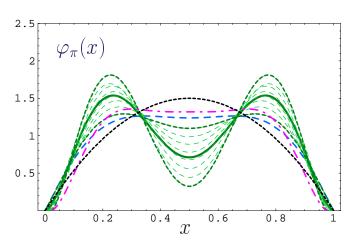


FIG. 1: Comparison of selected pion DAs denoted by obvious acronyms: $\varphi_{\rm as}$ (dotted line) [3, 6], $\varphi_{\rm PR}$ (dashed line) [20], $\varphi_{\rm Dor}$ (dot-dashed line) [21], and $\varphi_{\rm BMS}$ (solid line) [16]. Also shown is the whole "bunch" determined via QCD sum rules with non-local condensates [16]. All DAs are normalized at the same scale $\mu_0^2 \approx 1~{\rm GeV}^2$.

in different ways [8, 17]. However, on the grounds explained above, it appears that two-parameter models, the parameters being the first Gegenbauer coefficients a_2 and a_4 , enable one to fit all the moment constraints for $\langle \xi^N \rangle_{\pi}$, as well as to reproduce the value of $\langle x^{-1} \rangle_{\pi}$ within the QCD sum-rule error range, resulting into a "bunch" of DAs displayed in Fig. 1. The optimum sample out of this "bunch", termed BMS model, is described by the following expression

$$\varphi^{\text{BMS}}(x) = \varphi^{\text{as}}(x) \left[1 + a_2 C_2^{3/2} (2x - 1) + a_4 C_4^{3/2} (2x - 1) \right]$$
(2.5)

with $a_2 = +0.20$, $a_4 = -0.14$ and is emphasized by a solid line in Fig. 1. The shape of this "bunch" is confirmed by a

non-diagonal correlator, based on the QCD sum rules considered in [22].

III. CLEO DATA ANALYSIS

The CLEO data [15] on $F_{\pi\gamma}$ provide one rigorous constraint on theoretical models for the pion DA in QCD. Indeed, it was first shown in [23] that these data exclude the CZ pion DA because the prediction derived from it overshoots these data by orders of magnitude. Very recently, we analyzed [14, 19] the CLEO data by combining attributes from QCD light-cone sum rules [24, 25], NLO Efremov–Radyushkin–Brodsky–Lepage (ERBL) [2, 3, 6] evolution [26, 27], and detailed estimates of uncertainties owing to higher-twist contributions and NNLO perturbative corrections [28].

The upshot of this analysis is that the CZ pion DA is excluded at the 4σ level of accuracy and—perhaps somewhat surprisingly—that also the asymptotic pion DA

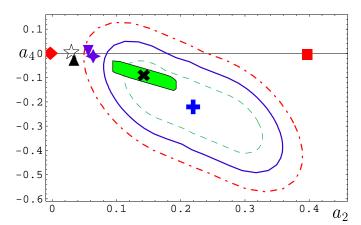


FIG. 2: Analysis of the CLEO data on $F_{\pi\gamma^*\gamma}(Q^2)$ in the $(a_2,\,a_4)$ plane in terms of error regions around the best-fit point (blue cross) with the following designations: 1σ (broken green line); 2σ (solid blue line); 3σ (dashed-dotted red line). Various theoretical models are also shown for comparison. The designations are as follows: -the asymptotic DA, -BMS model, -CZ DA, -best-fit point, -[29], -[20], -[31]-instanton models, and -transverse lattice result [30]. The slanted green rectangle represents the BMS "bunch" of pion DAs dictated by the nonlocal QCD sum rules for the value $\lambda_q^2 = 0.4 \; {\rm GeV^2}$. All constraints are evaluated at $\mu^2 = 5.76 \; {\rm GeV^2}$ after NLO ERBL evolution.

lies outside the 3σ error ellipse in the (a_2, a_4) plane (see Fig. 2), even if one allows the theoretical uncertainties owing to unknown higher-twist contributions to be of the order of 30% and presumes that the size of NNLO perturbative corrections is also large. On the other hand, the BMS pion DA calculated with a vacuum virtuality $\lambda_q^2 \simeq 0.4 \text{ GeV}^2$ was found to be inside the 1σ error ellipse, while other rival models, based on differing instantons approaches [20, 31], or derived with the aid of lattice simulations [30], are located in the vicinity of the border of the 3σ contour. It is worth emphasizing that the more precise the instanton-based models become, the further away from the asymptotic pion DA towards the region of the "bunch" they move (we refer to [19] for more details). It was pointed out before in [32] that the CLEO data ask for a broader pion DA than the asymptotic one.

In Fig. 3 (left panel) we compare our prediction for the scaled pion-photon

transition form factor with those from the CZ model (upper dashed line) and the asymptotic DA (lower dashed line). One observes that the strip obtained from the "bunch" of DAs is in very good agreement with both the CLEO data and also with the CELLO data [33]. The right panel of this figure illustrates in the form of a shaded band the region of uncertainty induced by our limited knowledge of higher-twist contributions. One observes that even the low- Q^2 CELLO data are in reasonable compliance with the theoretical prediction (the shaded strip).

Let us close this section by mentioning that other approaches claim to be able to describe the CLEO data with the asymptotic pion DA [23, 34, 35], taking into account only the leading-twist contribution and using only perturbative QCD (see for more details in [14]).

IV. OTHER EXCLUSIVE PROCESSES

Factorization theorems can be extended—at least formally—to baryons and their form factors [6]. The primary goal below is to give a brief summary of main results rather than to review the subject and the status of individual exclusive processes or baryon DAs (for a recent review, we refer to [36]). For instance, the situation concerning the nucleon DA is more

¹ The new model relative to [31], proposed in [21], involves more than two Gegenbauer coefficients and can therefore not be displayed in Fig. 2. However, reverting this model to an approximate one by utilizing only two (effective) Gegenbauer coefficients a_2 and a_4 shows that it is close to the 3σ error ellipse boundary, as said above.

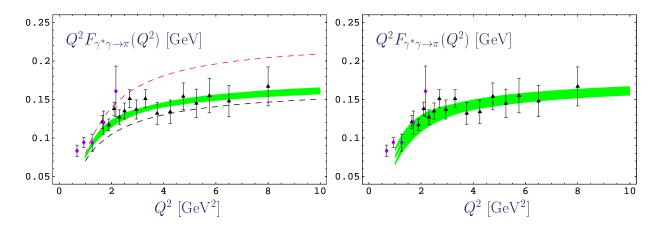


FIG. 3: Left: Light-cone sum-rule predictions for $Q^2F_{\gamma^*\gamma\to\pi}(Q^2)$ in comparison with the CELLO (diamonds, [33]) and the CLEO (triangles, [15]) experimental data evaluated with the twist-4 parameter value $\delta^2_{\text{Tw-4}} = 0.19 \text{ GeV}^2$ [14, 19]. The predictions correspond to selected pion DAs; viz., φ_{CZ} (upper dashed line) [11], BMS-"bunch" (shaded strip) [16], and φ_{as} (lower dashed line) [3, 6]. Right: Our prediction for $Q^2F_{\gamma^*\gamma\to\pi}(Q^2)$ corresponding to the "bunch" of pion DAs in Fig. 1 (shaded strip) in comparison with experimental data for twist-4 parameter values varied in the range $\delta^2_{\text{Tw-4}} = 0.15 - 0.23 \text{ GeV}^2$.

controversial compared to the meson case. It is undoubtedly true that the asymptotic nucleon DA is unable to describe the nucleon form factors [6]. On the other hand, asymmetric DAs constructed via moments determined by *local QCD* sum rules following [11], as, for example, in [37, 38, 39, 40, 41, 42] (see on the left part of Fig. 4 for an illustration), seem to yield to strongly suppressed results for the magnetic nucleon form factor when transverse momentum—intrinsic and Sudakov—effects are included [36, 43]. Valuable information on the inner structure of the nucleon was recently provided in [44] in the context of instantons, where it was shown that the shape of the proton DA is far from the asymptotic one.

While the nonperturbative nature of the nucleon is yet not well-understood, its evolution on the basis of the renormalization-group equation can be performed to a high level of accuracy within QCD perturbation theory. Indeed, within the basis of symmetrized Appell polynomials [36, 45], the nucleon evolution equation can be solved by employing factorization of the dependence on the longitudinal momentum from that on the external (large) momentum scale Q^2 up to any desired polynomial order.² The spectrum of the corresponding anomalous dimensions of trilinear quark operators was also determined [36, 45, 46] and its large-order behavior seems to increase logarithmically, reflecting the enhanced emission of soft gluons that forces the probability for finding bare quarks to decrease (see Fig. 4, right panel). This spectrum can be reproduced by the logarithmic fit

$$\gamma_n(M) = c + d\ln(M+b) . \tag{4.1}$$

The upper envelope of the spectrum is best described by the following values of the parameters with their errors: $b = 1.90989 \pm 0.00676$, $c = -0.637947 \pm 0.000634$, and $d = 0.88822 \pm 0.000119$. For the lower envelope, the corresponding values are $b = 3.006 \pm 0.483$, $c = -0.3954 \pm 0.0290$, and $d = 0.59691 \pm 0.00545$. The spacing of eigenvalues at very large order is reproduced by the values $b = -0.027 \pm 0.728$, $c = -0.2460 \pm 0.0248$, and $d = 0.291883 \pm 0.00475$. For

² The eigenfunctions of the nucleon evolution equation are linear combinations of symmetrized Appell polynomials, appropriately orthonormalized [36].

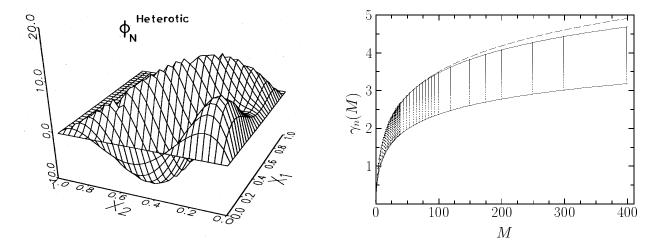


FIG. 4: Left: The heterotic nucleon distribution amplitude, proposed in [41]. Right: Spectrum of the anomalous dimensions of trilinear twist-3 quark operators up to order M=400. The solid lines (upper and lower envelopes of the spectrum) represent logarithmic fits up to the maximum considered order 400, taking into consideration all orders above 10. The dashed line gives for comparison a previous logarithmic fit [36] which takes into account all orders up to 150.

every order M, there are M+1 eigenfunctions of the same order with an excess of symmetric (under the permutation P_{13}) terms (denoted by black dots in Fig. 4) by one for even orders. The total number of eigenfunctions up to order M is $n_{\text{max}}(M) = \frac{1}{2}(M+1)(M+2)$ and the corresponding (M+1) eigenvalues are obtained by diagonalizing the $(M+1) \times (M+1)$ matrix. Up to order 150, both sectors (corresponding to the permutation parity $S_n = \pm 1$) of eigenvalues are included. Beyond that order, for reasons of technical convenience, only the antisymmetric (open circles) ones have been taken into account. The multiplet structure of the anomalous dimensions spectrum was found independently later on [47] in the context of a Hamiltonian approach to the one-dimensional XXX Heisenberg spin magnet of non-compact spin s=-1.

V. CONCLUDING REMARKS

Our discussion of the pion DA in the context of QCD sum rules with non-local condensates shows that the vacuum non-locality parameter can serve to extract valuable information on the underlying nonperturbative dynamics. The double-humped shape with suppressed endpoints of the derived pion DA is in good agreement with the CLEO data with a 1σ accuracy and agrees with the CELLO data as well. Progress of the non-local sum-rules approach to encompass tree-quarks states, like the nucleon, appears promising, while the perturbative apparatus for the evolution of such DAs is already well-developed.

In conclusion, let us mention as a personal statement that the major part of our scientific work depends to a great extent on the power of factorization theorems and their usage in QCD in the context of form factors, structure functions, etc. Therefore, we feel particularly attached to Prof. Efremov, given also that he was the Leader of the BLTPh QCD group, where two of us (A.P.B. and S.V.M.) have been working for over a decade, and he was also one of the opponents of one of us (N.G.S.) in defending his *Doctor fiziko-matematicheskih nauk* degree.

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